POWER SUPPLIES

It is usually the requirement of a power supply to provide a relatively ripple-free source of d-c potential from an a-c line. However, as seen in the last chapter, a rectifier actually provides an output which contains a-c components in addition to the d-c term that is desired, a measure of the a-c components being given by the ripple factor. It is customary to include a filter between the rectifier and the output to attenuate these ripple components. Often an electronic regulator is also included, if the regulation as well as the ripple must be small.

The analysis of the action of such rectifier filters is complicated by the fact that the rectifier as a driving source is nonlinear, thus requiring the solution of circuits with nonlinear elements. It is possible in most cases to make reasonable assumptions in order to effect an approximate engineering solution. In consequence, the results obtained are only approximate.

7-1. The Harmonic Components in Rectifier Circuits. An analytic representation of the output of the single-phase half-wave rectifier is obtained in terms of a Fourier series expansion. This series representation has the form

$$i = b_0 + \sum_{k=1}^{\infty} b_k \cos k\alpha + \sum_{k=1}^{\infty} a_k \sin k\alpha \tag{7-1}$$

where $\alpha = \omega t$ and where the coefficients that appear in the series are given by the integrals

$$\begin{aligned} b_0 &= \frac{1}{2\pi} \int_0^{2\pi} i \, d\alpha \\ b_k &= \frac{1}{\pi} \int_0^{2\pi} i \cos k\alpha \, d\alpha \\ a_k &= \frac{1}{\pi} \int_0^{2\pi} i \sin k\alpha \, d\alpha \end{aligned} \tag{7-2}$$

It should be recalled that the constant term b_0 that appears in this Fourier series is the average or d-c value of the current.

The explicit expression for the current in a half-wave rectifier circuit, which is obtained by performing the indicated integrations using Eqs.

(6-4) over the two specified intervals, yields

$$i = I_m \left[\frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \sum_{k=2,4,6,\ldots} \frac{\cos k\omega t}{(k+1)(k-1)} \right] \tag{7-3}$$

where $I_m = E_m/(r_p + R_l)$ and E_m is the peak transformer potential. The lowest angular frequency that is present in this expression is that of the primary source. Also, except for this single term of frequency ω, all other terms that appear in the expression are even-harmonic terms.

The corresponding Fourier series representation of the output of the full-wave rectifier which is illustrated in Fig. 6-5 may be derived from Eq. (7-3). Thus, by recalling that the full-wave circuit comprises two half-wave circuits which are so arranged that one circuit is operating during the interval when the other is not operating, then clearly the currents are functionally related by the expression $i_2(\alpha) = i_1(\alpha + \pi)$. The total load current, which is $i = i_1 + i_2$, then attains the form

$$i = I_m \left[\frac{2}{\pi} - \frac{4}{\pi} \sum_{k=2,4,6,\ldots} \frac{\cos k\omega t}{(k+1)(k-1)} \right] \tag{7-4}$$

where $I_m = E_m/(R_l + r_p)$, E_m being the maximum value of the transformer potential measured to the center tap.

A comparison of Eqs. (7-3) and (7-4) indicates that the fundamental angular-frequency term has been eliminated in the full-wave circuit, the lowest harmonic term in the output being 2ω, a second-harmonic term. This will be found to offer a distinct advantage in filtering.

The Fourier series representation of the half-wave and full-wave circuits using gas diodes can be obtained in the same way as above, although the form will be more complex. This is so because conduction begins at some small angle φ_0 and ceases at the angle $\pi - \varphi_0$, when it is assumed that the breakdown and the extinction potentials are equal. But since these angles are usually small under normal operating conditions, it will be assumed that Eqs. (7-3) and (7-4) are applicable for circuits with vacuum or gas diodes. The Fourier series representation of the output of a controlled rectifier is also possible, although the result is quite complex. However, such controlled rectifiers are ordinarily used in services in which the ripple is not of major concern, and, as a result, no detailed analysis will be undertaken. Some results will be given below covering these rectifiers, however.

7-2. Inductor Filters. The operation of an inductor filter depends on the inherent property of an inductor to oppose any change of current that may tend to take place through it. That is, the inductor stores energy in its magnetic field when the current is above its average value and releases energy when the current falls below this value. Consequently any sudden changes in current that might otherwise take place in the circuit will be smoothed out by the action of the inductor.

In particular, suppose that an inductor is connected in series with the load in a single-phase half-wave circuit, as illustrated in Fig. 7-1. For simplicity, suppose that the tube and choke resistances are negligible.

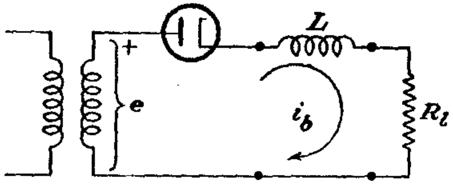

FIG. 7-1. Half-wave rectifier circuit with inductor filter.

Then the controlling differential equation for the current in the circuit during the time of current conduction is

$$L \frac{di_b}{dt} + R_l i_b = E_m \sin \omega t \tag{7-5}$$

A solution of this differential equation may be effected. This solution is complicated by the fact that current continues over only a portion of the cycle. The general character of the solution is shown graphically in Fig.

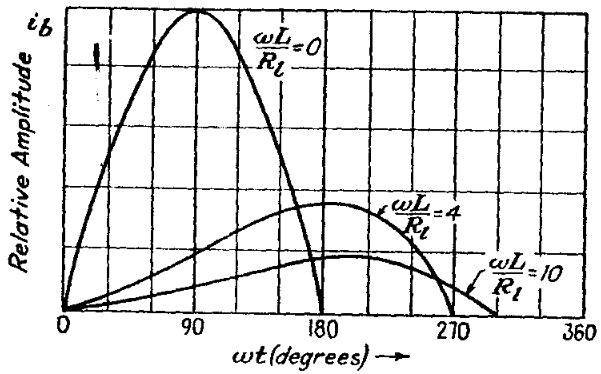

FIG. 7-2. The effect of changing inductance on the waveform of the current in a half-wave rectifier with inductor filter. The load R_l is assumed constant.

7-2, in which is shown the effect of changing the inductance on the waveform of the current. Since a simple inductance choke is seldom used with a half-wave circuit, further details of the analysis will not be given.

Suppose that an inductor filter is applied to the output of a full-wave rectifier. The circuit and a sketch of the output-current waveshape are given in Fig. 7-3. Since no cutout occurs in the current, the analysis assumes a different form from that for the half-wave case. Now, instead of considering the circuit differential equation, as in Eq. (7-5), and adjusting the initial conditions to fulfill the required physical conditions, an approximate solution is effected. It is supposed that the equation of the potential that is applied to the filter is given by Eq. (7-4). Moreover, it is noted that the amplitudes of the a-c terms beyond the first, and this is of second-harmonic frequency, are small compared with that of the first term. In particular, the fourth-harmonic frequency term is only 20 per cent of the second-harmonic term. Furthermore, the impedance of the inductor increases with frequency, and better filtering action exists for the higher-harmonic terms. Consequently it is assumed that all higher-order terms may be neglected.

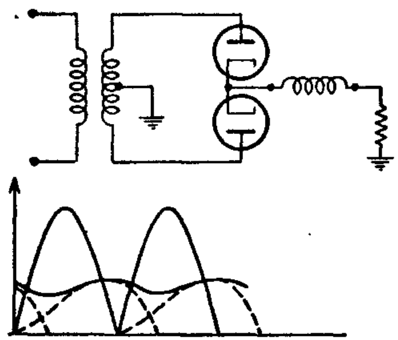

FIG. 7-3. Full-wave rectifier circuit with inductor filter, and the waveshape of the load current.

In accordance with the discussion, it is supposed that the input potential to the rectifier and load has the approximate form

$$e = \frac{2E_m}{\pi} - \frac{4E_m}{3\pi}\cos 2\omega t \qquad (7\text{-}6)$$

The corresponding load current is, in accordance with a-c circuit theory,

$$i_l = \frac{2E_m}{\pi R_l} - \frac{4E_m}{3\pi}\frac{\cos(2\omega t - \psi)}{\sqrt{R_l^2 + 4\omega^2 L^2}} \qquad (7\text{-}7)$$

where

$$\tan\psi = \frac{2\omega L}{R_l} \qquad (7\text{-}8)$$

The ripple factor, defined in Eq. (6-14), becomes

$$r = \frac{(4E_m/3\pi)(1/\sqrt{R_l^2 + 4\omega^2L^2})}{2E_m/\pi R_l} = \frac{2R_l}{3\sqrt{2}}\frac{1}{\sqrt{R_l^2 + 4\omega^2L^2}}$$

which may be expressed in the form

$$r = \frac{2}{3\sqrt{2}}\frac{1}{\sqrt{1 + (4\omega^2L^2/R_l^2)}} \qquad (7\text{-}9)$$

If the ratio $\omega L/R_l$ is large, this reduces to

$$r = \frac{1}{3\sqrt{2}}\frac{R_l}{\omega L} \qquad (7\text{-}10)$$

This expression shows that the filtering improves with decreased load resistance or, correspondingly, with increased load current. At no load, $R_l = \infty$, and the filtering is poorest, with $r = 2/3\sqrt{2} = 0.47$. This is also the result which applies when no choke is included in the circuit. [Compare with result with Eq. (6-21), which gives 0.482. The difference arises from the terms in the Fourier series that have been neglected.] The expression also shows that large inductances are accompanied by decreased ripple.

The d-c output potential is given by

$$E_{d\text{-}c} = I_{d\text{-}c}R_l = \frac{2E_m}{\pi} = 0.637E_m = 0.90E_{rms} \qquad (7\text{-}11)$$

where E_{rms} is the transformer secondary potential measured to the center tap. Note that under the assumptions made, viz., negligible power-transformer leakage reactance, transformer resistance, tube resistance, and inductor resistance, the output potential does not change with load, with consequent perfect regulation. Because the neglected effects are not negligible, the output potential actually decreases with increased current.

7-3. Capacitor Filter. Filtering is frequently effected by shunting the load with a capacitor. During the time that the rectifier output is increasing, the capacitor is charging to the rectifier output potential and energy is stored in the capacitor. During the time that the rectifier potential falls below that of the capacitor, the capacitor delivers energy to the load, thus maintaining the potential at a high level for a longer period than without the capacitor. The ripple is therefore considerably decreased. Clearly, the diode acts as a switch, permitting charge to flow into the capacitor when the rectifier potential exceeds the capacitor potential, and then acts to disconnect the power source when the potential falls below that of the capacitor.

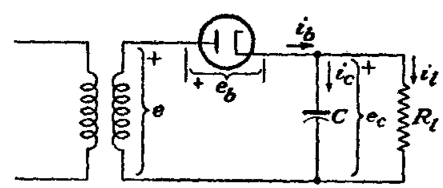

FIG. 7-4. A single-phase half-wave capacitor-filtered rectifier.

To examine the operation in some detail, refer to Fig. 7-4, which shows a diagram of the circuit. The tube current during the conducting portion of the cycle is

$$i_b = i_C + i_l \qquad (7\text{-}12)$$

where

$$i_l = \frac{e_l}{R_l} = \frac{e_C}{R_l} \qquad (7\text{-}13)$$

and where

$$i_C = \frac{dq_C}{dt} = C\frac{de_C}{dt} \qquad (7\text{-}14)$$

where q_C is the capacitor charge. The controlling differential equation of the charging current through the tube is then

$$i_b = \frac{e_C}{R_l} + C\frac{de_C}{dt} \tag{7-15}$$

But the potential e_C during the time that the tube is conducting is simply the transformer potential, if the tube drop is neglected. Hence the capacitor potential during this portion of the cycle is sinusoidal and is

$$e_C = e = E_m \sin \omega t$$

The corresponding tube current is

$$i_b = \frac{E_m}{R_l}\sin \omega t + \omega C E_m \cos \omega t$$

This may be written in the equivalent form

$$i_b = E_m\sqrt{\omega^2C^2 + \frac{1}{R_l^2}}\sin(\omega t + \psi) \tag{7-16}$$

where

$$\psi = \tan^{-1}\omega C R_l \tag{7-17}$$

A sketch of the current wave is illustrated in Fig. 7-5.

Equation (7-16) shows that the use of large capacitances, in order to improve the filtering, is accompanied by large tube currents. Therefore,

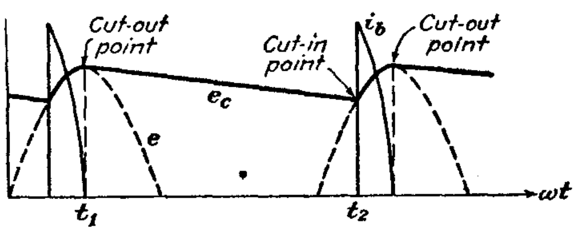

FIG. 7-5. The tube current and the load potential in a single-phase half-wave capacitor-filtered rectifier.

if a large capacitance is used for a given load in order to maintain the output potential more nearly constant, a very peaked current exists. In fact, for a certain required average current demand by the load, the tube-current pulse becomes more and more peaked as the capacitance is made larger. This imposes serious duty conditions on the tube, since the average current through the tube may be well within the tube rating and yet the large peak current may injure the cathode. Vacuum diodes would not be appreciably damaged by the high peak-current demands, since temperature-saturated currents may be drawn without seriously injuring the cathode. In the case of gas tubes, however, any attempt to draw higher than temperature-saturated current will usually be accompanied by severe positive-ion bombardment of the cathode, with consequent cathode disintegration. It is for this reason that large-capacitance input filters should not be used with rectifiers that employ gas diodes.

When the tube stops conducting, $i_b = 0$ and the controlling differential equation during the nonconducting portion of the cycle is, from Eq. (7-15),

$$C\frac{de_C}{dt} + \frac{e_C}{R_l} = 0 \tag{7-18}$$

The solution of this differential equation is

$$e_C = Ae^{-t/R_lC} \tag{7-19}$$

This shows that the capacitor discharges exponentially through the load.

To determine the value of the constant A that appears in this expression, use is made of the fact that at the time $t = t_1$, the cutout time,

$$e_C = e = E_m \sin \omega t_1$$

Combining this result with Eq. (7-19) gives

$$A = E_m \sin \omega t_1 e^{t_1/R_lC} \tag{7-20}$$

and Eq. (7-19) becomes

$$e_C = E_m \sin \omega t_1 e^{-(t-t_1)/R_lC} \tag{7-21}$$

The quantity t_1 that appears in this expression is known, since at $t = t_1$ the tube current is zero. From Eq. (7-16) this requires

$$\sin(\omega t_1 + \psi) = 0$$

from which it follows that

$$\omega t_1 = \pi - \psi = \pi - \tan^{-1}\omega C R_l \tag{7-22}$$

If t_1 from Eq. (7-22) is substituted in Eq. (7-21), there results

$$e_C = E_m \sin \omega t_1 e^{-(\omega t+\psi-\pi)/\omega C R_l} \tag{7-23}$$

To find the "cut-in" point, it is noted that e_C equals the impressed transformer potential e at this point. This requires

$$E_m \sin \omega t_2 = E_m \sin \omega t_1 e^{-(\omega t_2-\psi-\pi)/\omega C R_l}$$

or

$$\sin \omega t_2 = \sin \omega t_1 e^{-(\omega t_2-\psi-\pi)/\omega C R_l} \tag{7-24}$$

The evaluation of the cut-in time t_2 cannot be solved explicitly, for this is a transcendental equation. Graphical methods can be used effectively in this evaluation. The results are given in Fig. 7-6. Included on this graph are a plot of Eq. (7-22) for the cutout angle and a plot of Eq. (7-24) for the cut-in angle.

The foregoing analysis gives a complete specification of the operation of the capacitor filter, the current through the tube being given by Eqs. (7-16) and (7-17), the potential across the load resistor being given by

$$e_C = E_m \sin \omega t \qquad \text{for } \omega t_2 < \omega t < \omega t_1 \tag{7-25a}$$

and by Eq. (7-23)

$$e_C = E_m \sin \omega t_1 e^{-(\omega t - \omega t_1)/\omega R_l C} \qquad \text{for } \omega t_1 < \omega t < 2\pi + \omega t_2 \tag{7-25b}$$

With this information it is possible to evaluate the d-c output potential, the ripple factor, the peak tube current, etc. These quantities may then be plotted as functions of the parameters R_l, C, E_m. Such an analysis is quite involved, but it has been carried out,[1] and the results are given in graphical form.

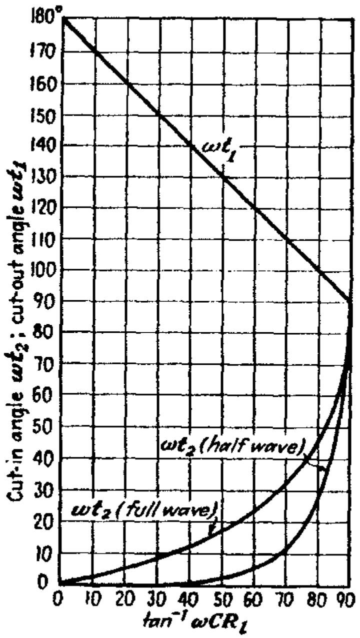

Fig. 7-6. Plot of cut-in angle ωt_2 and cutout angle ωt_1 vs. circuit parameters for the capacitor filter.

7-4. Approximate Analysis of Capacitor Filters. It is expedient to make several reasonable approximations in order to obtain an approximate analysis of the behavior of the capacitor filter. Such an approximate analysis possesses the advantage that the important factors of the operation are simply related to the circuit parameters. Moreover, the results are sufficiently accurate for most engineering applications. The character of the approximation is made evident by an inspection of Fig. 7-7, which shows the trace of an oscillogram of the load potential in a single-phase full-wave capacitor-filtered rectifier. The potential curve may be approximated by two straight-line segments, as shown in Fig. 7-8. If the total capacitor discharge potential is denoted as E_r, then, from the diagram, the average value of the potential is

$$E_{d\text{-}c} = E_m - \frac{E_r}{2} \tag{7-26}$$

Also, the rms value of the triangular ripple potential may be shown to be

$$E'_{\text{rms}} = \frac{E_r}{2\sqrt{3}} \tag{7-27}$$

Also, if it is assumed that the capacitor discharge continues for the full half cycle at a constant rate which is equal to the average load current $I_{d\text{-}c}$, the fall in potential during this half cycle is E_r. That is, approximately

$$E_r = \frac{I_{d\text{-}c}}{2fC} \tag{7-28}$$

The ripple factor is then given by

$$r = \frac{E'_{\text{rms}}}{E_{d\text{-}c}} = \frac{E_r}{2\sqrt{3}\,E_{d\text{-}c}} = \frac{I_{d\text{-}c}}{4\sqrt{3}\,fCE_{d\text{-}c}}$$

But since $E_{d\text{-}c} = I_{d\text{-}c}R_l$,

$$r = \frac{1}{4\sqrt{3}\,fCR_l} \tag{7-29}$$

This expression shows that the ripple factor varies inversely with the load resistance and the filter capacitance. At no load, $R_l = \infty$, and the ripple is zero. As R_l decreases, corresponding to increasing current, the ripple becomes larger. Also, for given R_l, the ripple is smaller for large capacitances. Actually, Eq. (7-29) is more nearly correct for small values of ripple than for the larger values, the value of ripple being generally larger than that obtained experimentally. The results are adequate for most purposes.

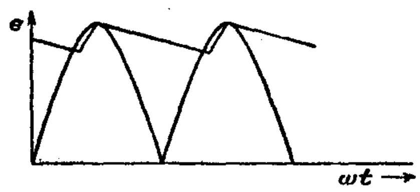

Fig. 7-7. Oscillogram of the load potential in a single-phase full-wave capacitor-filtered rectifier.

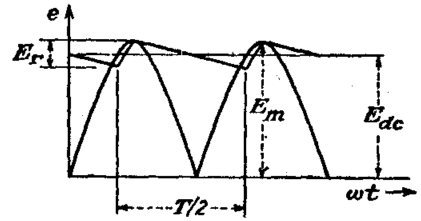

Fig. 7-8. The approximate load-potential waveform corresponding to the curves of Fig. 7-7.

The regulation curve is obtained by combining Eqs. (7-26) and (7-28). This yields

$$E_{d\text{-}c} = E_m - \frac{I_{d\text{-}c}}{4fC} \tag{7-30}$$

This expression represents a linear fall in potential with d-c output current. Also, it shows that the simple capacitor filter will possess poor regulation unless the capacitance C is large.

Now refer to the circuit of Fig. 7-4 to ascertain the peak inverse potential across the tube. It is seen to be twice the transformer peak potential. For the full-wave case, the peak inverse potential is also twice the transformer maximum potential, as measured from the mid-point to either end, or the full transformer potential. Thus the presence of the capacitor increases the peak inverse potential in the half-wave circuit from E_m to $2E_m$ but does not affect the peak inverse potential in the full-wave circuit.

7-5. *L*-section Filter. An *L*-section filter consists of a series inductor and a shunt capacitor, as shown in Fig. 7-9. This filter is so arranged that the inductor offers a high impedance to the harmonic terms, and the capacitor shunts the load, so as to bypass the harmonic currents. The resulting ripple is markedly reduced over that of the relatively simple filters of Secs. 7-2 and 7-3.

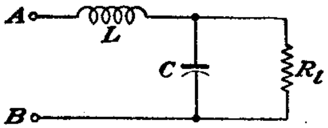

Fig. 7-9. An *L*-section filter.

The ripple factor is readily approximated by taking for the potential applied to the input terminals of the filter the first two terms in the Fourier series representation of the output potential of the rectifier, viz.,

$$e = \frac{2E_m}{\pi} - \frac{4E_m}{3\pi} \cos 2\omega t \tag{7-31}$$

But since the filter elements are chosen to provide a high series impedance and a very low shunting impedance, certain plausible approximations may be made. Thus, since the choke impedance is high compared with the effective parallel impedance of the capacitor and load resistor, the net impedance between terminals *AB* is approximately X_L and the a-c current through the circuit is

$$I'_{\text{rms}} \doteq \frac{4E_m}{3\sqrt{2}\,\pi} \frac{1}{X_L} = \frac{\sqrt{2}}{3} E_{\text{d-c}} \frac{1}{X_L} \tag{7-32}$$

Likewise, since the a-c impedance of the capacitor is small compared with R_l, it may be assumed that all the a-c current passes through the capacitor and none through the resistor. The a-c potential across the load (the ripple potential) is the potential across the capacitor and is

$$E'_{\text{rms}} \doteq \frac{\sqrt{2}}{3} E_{\text{d-c}} \frac{X_C}{X_L} \tag{7-33}$$

The ripple factor is then given by

$$r = \frac{\sqrt{2}}{3} \frac{X_C}{X_L} = \frac{\sqrt{2}}{3} \frac{1}{2\omega C} \frac{1}{2\omega L} \tag{7-34}$$

which may be written, at 60 cps, with *L* in henrys and *C* in microfarads,

$$r = \frac{0.830}{LC} \tag{7-35}$$

It should be noted that the effect of combining the decreasing ripple of the inductor filter and the increasing ripple of the simple capacitor filter for increasing loads is a constant ripple circuit, independent of load.

The above analysis assumes that no current cutout exists at any time of the cycle. If it did, the analysis would follow along the lines of Sec. 7-3 and Eq. (7-31) for the potential would not apply. But since with no

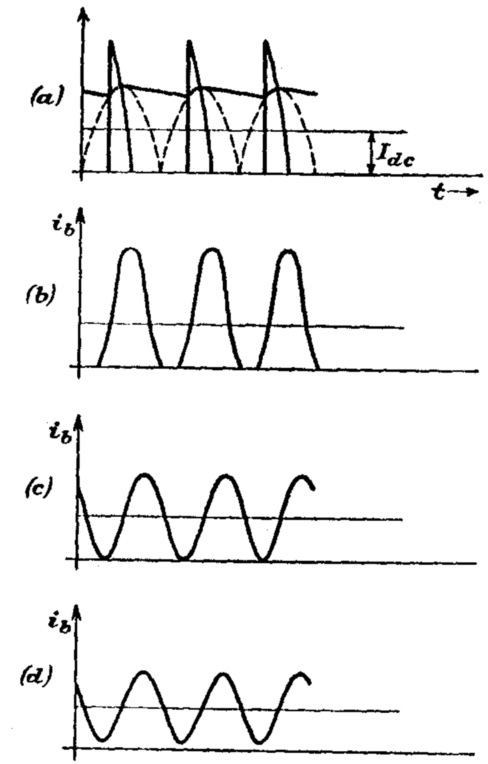

Fig. 7-10. The tube-current waveform in the full-wave rectifier with an *L*-section filter, when (*a*) $L = 0$, (*b*) $L < L_c$, (*c*) $L = L_c$, (*d*) $L > L_c$, for constant $I_{d\text{-}c}$.

inductance in the filter cutout will occur, whereas with sufficient inductance there will be no cutout, it would be expected that there would be some minimum inductance for a given current below which cutout would occur, although for larger values than this critical value the conduction would continue for the full cycle. The situation is best illustrated graphically. Figure 7-10 shows the expected tube current for various amounts of series inductance *L*.

If the rectifier is to pass current throughout the entire cycle, the peak current delivered must not exceed the d-c component. But the d-c value is $E_{\text{d-c}}/R_l$. Also, the peak a-c current is $(2E_{\text{d-c}}/3)(1/X_L)$. Hence for current flow during the full cycle it is necessary that

$$\frac{E_{d\text{-}c}}{R_l} \geqq \frac{2E_{d\text{-}c}}{3}\frac{1}{X_L}$$

or

$$X_L \geqq \frac{2R_l}{3} \tag{7-36}$$

from which the value for the critical inductance is found to be

$$L_c = \frac{2R_l}{3\omega}$$

which has the value

$$L_c = \frac{R_l}{1{,}130} \tag{7-37}$$

for a 60-cps power frequency, where R_l is in ohms and L_c is in henrys. However, owing to the approximations that have been made in this analysis, it is advisable for conservative design to use a larger value of L_c than that given in Eq. (7-37). A good practical figure to choose as the denominator is 1,000 instead of 1,130.

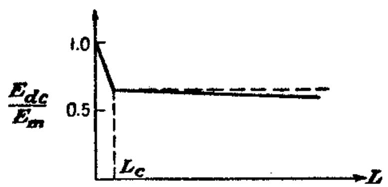

FIG. 7-11. The regulation curve of a rectifier with L-section filter as a function of series inductance, for constant output current.

The effect of the cutout is illustrated in Fig. 7-11, which shows a regulation curve of the system, for constant R_l and varying series inductance. Clearly, when the series inductance is zero, the filter is of the simple capacitance type and the output potential is approximately E_m. With increasing inductance, the potential falls, until at $L = L_c$ the output potential is that corresponding to the simple L filter with no cutout, or $0.637E_m$. For values of L greater than L_c, there is no change in potential, except for the effects of the resistances of the various elements of the circuit.

It is not possible to satisfy the conditions of Eq. (7-37) for all values of load, since at no load this would require an infinite inductance. However, when good potential regulation is desired, it is customary to use a bleeder resistance across the load so as to maintain the conditions of Eq. (7-37) even if this represents a power loss.

A more efficient method than using a high bleeder current, with its attendant power dissipation, is to make use of the fact that the inductance of an iron-core reactor depends, among other things, on the amount of d-c current in the winding. Chokes for which the inductance is high at low values of d-c current and which decrease markedly with increased d-c currents are called "swinging" chokes. The swinging choke is provided with a closed iron core, whereas the core of the inductor which is to possess a more nearly constant inductance is provided with a narrow air gap. A typical curve for such a reactor is illustrated in Fig. 7-12. The advantage of such a choke is that for high R_l, and therefore low d-c current, the inductance is high. As a result, the conditional equation (7-37) is satisfied over a wider range of R_l. Clearly, however, when a swinging choke is used, the ripple factor is no longer independent of the load.

The above analysis for the critical inductance of the L-type filter applies for the full-wave rectifier for which

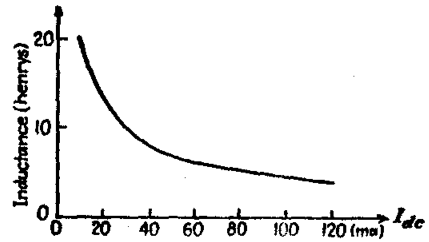

FIG. 7-12. The inductance of a swinging choke as a function of the d-c current through it.

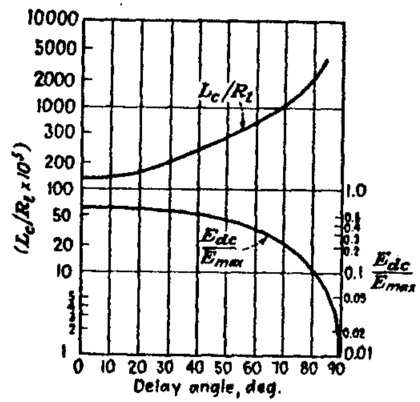

FIG. 7-13. Critical inductance and d-c output potential as a function of the delay angle in a full-wave controlled rectifier.

conduction continues for 180 deg in each cycle. Consequently the results so obtained are not applicable when an L-section filter is used with a controlled rectifier. The analysis for a full-wave controlled rectifier is considerably more complicated than that above, owing to the fact that the amplitude of the harmonics in the Fourier series representation of the output depends on the delay angles, and these are of such amplitude that they cannot be neglected in the analysis. The results of such an analysis are given graphically[2] in Fig. 7-13. The curves give a measure of both the critical inductance and the output potential.

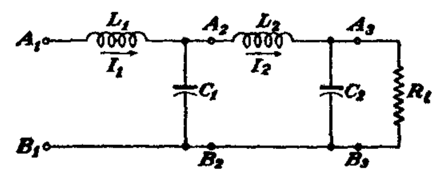

FIG. 7-14. A two-unit L-section filter.

7-6. Multiple L-section Filters. If it is desired to limit the ripple to a value that is less than that possible[4] with a single L-section filter using commercially available elements, two or more L-section filters may be connected in cascade, as shown in Fig. 7-14. An approximate solution is possible by following the methods of Sec. 7-5. It is assumed, therefore, that the choke impedances are much larger than the reactances of the capacitors. Also, it is assumed that the reactance of the last capacitor is small compared with the resistance of the load. Under these assumptions, the impedance between A_3 and B_3 is X_{C2}. The impedance between A_2 and B_2 is X_{C1}, and the impedance between A_1 and B_1 is X_{L1}, approximately.

The a-c current I_1 is approximately

$$I_1 = \frac{\sqrt{2}}{3} E_{d\text{-}c} \frac{1}{X_{L1}}$$

The a-c potential across C_1 is approximately

$$E_{A_2B_2} = I_1 X_{C1}$$

The a-c current I_2 is approximately

$$I_2 = \frac{E_{A_2B_2}}{X_{L2}}$$

The a-c potential across the load is approximately

$$I_2 X_{C2} = I_1 X_{C2} \frac{X_{C1}}{X_{L1}} = \frac{\sqrt{2}}{3} E_{d\text{-}c} \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}}$$

The ripple factor is given by the expression

$$r = \frac{\sqrt{2}}{3} \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}} \tag{7-38}$$

A comparison of this expression with Eq. (7-34) indicates the generalization that should be made in obtaining an expression for the ripple factor of a cascaded filter of n sections. The expression would have the form

$$r = \frac{\sqrt{2}}{3} \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}} \cdots \frac{X_{Cn}}{X_{Ln}} \tag{7-39}$$

If the sections are all similar, then Eq. (7-39) becomes

$$r = \frac{\sqrt{2}}{3} \left(\frac{X_C}{X_L}\right)^n = \frac{\sqrt{2}}{3} \frac{1}{(16\pi^2 f^2 LC)^n} \tag{7-40}$$

where f is the source frequency. It follows from this that the required LC product for a specified ripple factor r at a 60-cps source frequency is given by

$$LC = 1.76 \left(\frac{0.471}{r}\right)^{1/n} \tag{7-41}$$

Note also that, to the approximation that the impedance between A_2 and B_2 is simply X_{C1}, the critical inductance is given by Eq. (7-37), as for the single-section unit.

7-7. Π-section Filter. The use of a Π-section filter provides an output potential that approaches the peak value of the a-c potential of the source, the ripple components being very small. Such a filter is illustrated in Fig. 7-15. Although such filters do provide a higher d-c output potential than is possible with an L-section filter, the tube currents are peaked and the regulation is generally poor, these results being common with the simple capacitor filter.

A study of the oscilloscope patterns at various points of such a filter shows that the action can be understood by considering the inductor and the second capacitor as an L-section filter that acts on the triangular output potential wave from the first capacitor. The output potential is then approximately that from the input capacitor, the ripple contained in this output being reduced by the L-section filter. That is, the ripple factor of the Π-section filter is given approximately by

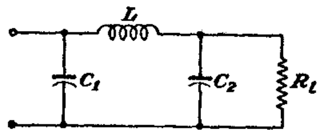

Fig. 7-15. A Π-section filter.

$$r_\pi = r_C r_L \tag{7-42}$$

where r_C is given by Eq. (7-29) and r_L is given by Eq. (7-34). This becomes

$$r_\pi = 0.855 \frac{X_{C1} X_{C2}}{X_L R_l} \tag{7-43}$$

with all reactances calculated at the second-harmonic frequency. For a 60-cps power source, this is

$$r_\pi = \frac{2 \times 10^3}{C_1 C_2 L R_l} \tag{7-44}$$

with the capacitances in microfarads, the inductance in henrys, and the resistance in ohms. This result is only approximate, since it assumes in effect that the ripple output from the capacitor filter is sinusoidal rather than triangular.

A somewhat more accurate evaluation of the ripple factor, due to Arguimbau,[3] is possible. The technique employed is similar to that used to evaluate the grid driving power of a class C amplifier. For the filter connected to a rectifier at the power frequency ω, the important ripple term is of second-harmonic frequency. Consequently, it is required to find the peak value of the second-harmonic current I'_{2m} to the input capacitor of the Π filter. This is given by the Fourier component

$$I'_{2m} = \frac{1}{\pi} \int_0^{2\pi} i_b \cos 2\omega t \, d(\omega t) \tag{7-45}$$

Now assume that the current pulse is significant only near the peak value of the cosine curve. Therefore, the cos $2\omega t$ factor appearing in the integral is replaced by unity, and approximately

$$\sqrt{2}\, I_2' \doteq \frac{1}{\pi} \int_0^{2\pi} i_b\, d(\omega t) = 2 I_{d\text{-}c} \tag{7-46}$$

Hence, the upper limit of the rms second-harmonic potential is

$$E_2' = I_2' X_{C1} = \sqrt{2}\, I_{d\text{-}c} X_{C1} \tag{7-47}$$

But the potential E_2' is applied to the L section, so that, by the same logic as before, the output ripple is $E_2' X_{C2}/X_L$. Hence, the ripple factor is

$$r_\pi = \sqrt{2}\, \frac{I_{d\text{-}c} X_{C1} X_{C2}}{E_{d\text{-}c} X_L} = \sqrt{2}\, \frac{X_{C1}}{R_l} \frac{X_{C2}}{X_L} \tag{7-48}$$

where all reactances are calculated at the second-harmonic frequency. At 60-cps primary frequency, this reduces to

$$r_\pi = \frac{3.3 \times 10^3}{C_1 C_2 L R_l} \tag{7-49}$$

Note that here, as in the previous analysis, the effects of higher harmonics than the second have been neglected. This result is probably more accurate than that given in Eq. (7-44) owing to the more reasonable approximation in the analysis.

If the inductor of the Π-section filter is replaced by a resistor, a practice that is often used with low-current-drain power supplies, the ripple factor is given by Eq. (7-48) with X_L replaced by R. Thus

$$r_\pi = \sqrt{2}\, \frac{X_{C1} X_{C2}}{R R_l}$$

or

$$r_\pi = \frac{2.5 \times 10^6}{C_1 C_2 R R_l} \tag{7-50}$$

7-8. Glow-tube Regulator. The use of an electronic rectifier with an appropriate filter serves to provide a low-ripple source of d-c potential, the percentage of ripple present in the output depending upon the form of the filter that is used. Such rectifier systems, while generally satisfactory for many purposes, possess several shortcomings which may make them inadequate for certain services. The output potential depends critically upon the input potential to the rectifier, and a poorly regulated power system will be accompanied by a corresponding change in the output from the rectifier. Also, since the output impedance of the rectifier is usually quite high, the rectifier system will possess a poor regulation.

It is frequently necessary to construct a power supply the output potential of which is constant over wide ranges of input a-c potential, so as to provide a constant output potential source from a poorly regulated power line. Or it may be necessary to maintain a constant output potential for a varying output load. Electronic potential regulators provide such a control device and are extensively used for such service.

The simplest form of potential regulator makes use of the substantially constant potential characteristic of a glow tube. A glow tube is a cold-cathode discharge tube which is characterized by a fairly high tube drop

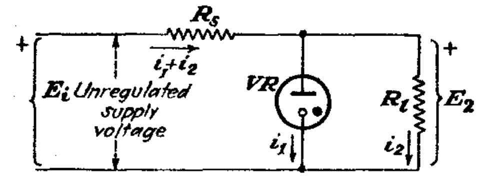

FIG. 7-16. Glow-tube potential regulator.

and a low current-carrying capacity (see Sec. 1-29). The potential across the tube over the operating range is fairly constant and independent of the current. When connected in the circuit shown in Fig. 7-16, the potential across the load will be a constant and equal to the tube drop of the glow tube, over a range of currents. Specifically, if a VR-150/30 is used, the potential across the load will be approximately 150 volts provided that the current through the tube does not exceed the rated 30 ma of the tube.

If a potential is desired that is higher than that of a single glow tube, several tubes may be connected in series. This will provide a constant potential source that is the sum of the tube drops of the tubes that are used. For example, the use of a VR-150 and a VR-105 in series will provide a constant 255-volt source. The supply potential must be greater than the breakdown potential of the tubes in order to make operation possible. The difference between the supply potential and the operating tube potential drop will appear across the stabilizing resistor R_s.

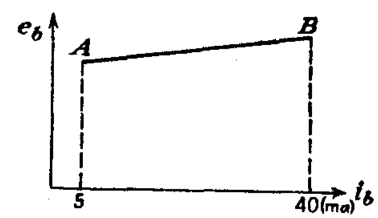

FIG. 7-17. A typical glow-tube volt-ampere characteristic.

An analysis of such a circuit is readily possible[4] if use is made of the practical fact that over the range of operation the volt-ampere characteristic of the regulator tube is almost linear. A typical characteristic has the form shown in Fig. 7-17 (compare with Fig. 1-37). This characteristic may be expressed by an equation of the form

$$e = a i_1 + b \tag{7-51}$$

where

$$a = \frac{e_B - e_A}{i_B - i_A} \qquad \text{and} \qquad b = e_A - a i_A \tag{7-52}$$

From the circuit diagram of Fig. 7-16 it is seen that

$$E_i = (i_1 + i_2)R_s + E_b \tag{7-53}$$

Also,

$$E_b = E_2 = R_l i_2 = a i_1 + b \tag{7-54}$$

from which it follows that

$$i_1 = \frac{E_2 - b}{a}$$

and

$$i_2 = \frac{E_2}{R_l} \tag{7-55}$$

Combining these equations yields

$$E_i = \left(\frac{E_2 - b}{a} + \frac{E_2}{R_l}\right) R_s + E_2$$

from which

$$E_2 = \frac{a R_l E_i + b R_l R_s}{R_l R_s + a R_s + a R_l} \tag{7-56}$$

This is the expression for the regulated potential as a function of the supply potential and the circuit parameters.

The variation of the output potential as the input potential varies is of considerable importance. The ratio dE_i/dE_2 is known as the stabilization ratio and is found to be

$$S = \frac{dE_i}{dE_2} = \frac{R_l(R_s + a) + aR_s}{aR_l} \tag{7-57}$$

Combining this with the expression for E_2 gives

$$S = \frac{aE_i + bR_s}{aE_2} \tag{7-58}$$

This equation shows that for perfect regulation, i.e., infinite stabilization ratio, the fraction dE_i/dE_2 should be infinite. For best stabilization features, both E_2 and a should be small; and b, E_i, and R_s should be large. Using typical values with a VR-75 tube, one finds

$E_i = 250$ volts

$R_l = \infty$	$R_s = 32.5$ kilohms	$\Delta E_i = 20$	$\Delta E_2 = 0.15$
$R_l = 3$ kilohms	$R_s = 5.85$ kilohms	$\Delta E_i = 20$	$\Delta E_2 = 0.60$

Such simple gas-tube regulators operate quite satisfactorily, but they are seriously limited in their usefulness because of their limited flexibility, both because of their fixed potential ratings and the relatively low current-carrying capacity.

7-9. Simple Vacuum-tube Regulator. One may employ the variable beam-resistance characteristic of a vacuum tube to maintain the output potential from a power supply at a substantially constant level. The beam resistance, as previously defined, is $r_b = e_b/i_b$ and is quite different from the plate resistance r_p of the tube. The circuit of such a simple potential regulator is given in Fig. 7-18.

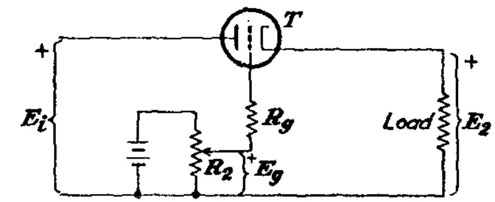

FIG. 7-18. A simple vacuum-tube potential regulator.

Assume that the potential across the load is at the desired value. Under this condition, the cathode is positive relative to ground by a potential E_2. The grid may be made positive relative to ground by a potential E_g which is less than E_2. The potentiometer R_2 is adjusted until the bias on the tube is such that the tube will pass the requisite load current. With this bias, the resistance of T is established at the desired value to reduce the rectifier output to the desired load potential.

If the rectifier output potential increases for whatever reason, the potential at the cathode of T tends to increase. As E_2 increases, the bias on the tube increases and the effective beam resistance of the tube becomes greater. Consequently the potential drop across the tube becomes greater. If the circuit is properly designed, the increased potential across T is approximately equal to the increase of rectifier output potential and the rectifier output potential remains substantially constant.

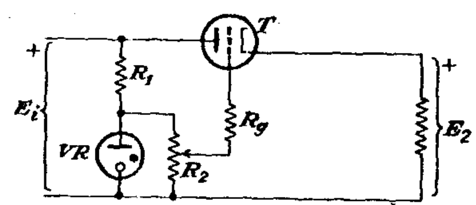

FIG. 7-19. A simple potential-regulator circuit.

The practical form of the circuit will replace the battery by a glow tube. Such a circuit is shown in Fig. 7-19.

The output potential from this regulator is not absolutely constant, since for an increased input to the circuit the potential at the cathode of T must rise slightly if the regulator is to function. However, if the characteristics of tube T are carefully chosen, the rise of load potential is not large.

It would be expected, of course, that vacuum tubes in which the beam resistance varied rapidly with small changes in bias would be most desirable for service in such regulators. Tubes possessing such a characteristic could probably be designed if there were no alternative approach. Actually such special tubes are not necessary, as it is quite possible to achieve the same ends by including a d-c amplifier in the circuit in such a way that slight changes in output potential are amplified before being applied degeneratively to T. These circuits are used extensively, and a detailed analysis will be given below.

7-10. Electronic Potential Regulators—Basic Considerations.[5] The design of electronically regulated power supplies has become fairly well standardized. The elements of such a circuit are given in Fig. 7-20.

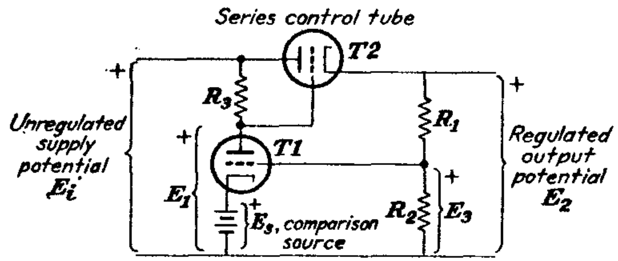

FIG. 7-20. A basic electronic regulator circuit.

As discussed above, the operation of the circuit is essentially the following: The source of unregulated potential from the rectifier and filter is applied across the input terminals of the regulator. The unregulated d-c current is fed through the series control tube, now designated $T2$ in the diagram, to the output circuit. The regulating action is obtained by comparing a fixed fraction of the output potential with a standard potential source, such as a battery or a VR gas tube. Any difference between the two is applied degeneratively after amplification by a high-gain d-c amplifier to the control grid of the series current-control tube. The correction may be made nearly perfect by using a d-c amplifier of sufficient gain. If the current requirements are too high for a single tube, a number of such tubes may be connected in parallel. A power triode is frequently used for this purpose, although a pentode or beam tetrode connected as a triode will serve equally well. Popular series control tubes include 6AS7G, 6B4, 6L6, 6V6, and 6Y6G. These tubes will pass approximately 75 ma without seriously exceeding the plate dissipation of the tube.

The design of a potential regulator requires a knowledge of the characteristic curves of the control tube and also of the d-c amplifier tube. However, since small changes in potential and current are ordinarily involved, the circuit operation may be analyzed in terms of the slope of the tube characteristics—specifically, in terms of the mutual conductance and the internal resistance of the tubes at the operating points. Several cases of interest will be examined separately.

a. Varying Input Potential—R_3 Connected to the Input Side of the Regulator. Suppose that the output load remains constant but that the input potential to the regulator varies, either because of the poor regulation of the input a-c supply potential to the rectifier or because of the ripple in the output of the rectifier due to inadequate filtering. It is desired to determine the change in output potential under these conditions. This may be done by examining the change in potential across $T2$ due to the change in input potential.

Consider the changes that occur in $T2$ due to changes in electrode potentials. For small variations about the levels specified in Fig. 7-20,

$$i_{p2} = g_{m2}e_{g2} + \frac{1}{r_{p2}} e_{p2} \tag{7-59}$$

Note, however, that

$$e_{g2} = dE_1 - dE_2$$

and

$$e_{p2} = dE_i - dE_2 \tag{7-60}$$

Note also that

$$i_{p2} = \frac{dE_2}{R_l}$$

where R_l is the equivalent load resistance, which is assumed constant. By combining these equations, there follows

$$\frac{dE_2}{R_l} = g_{m2}(dE_1 - dE_2) + \frac{1}{r_{p2}}(dE_i - dE_2)$$

which becomes

$$dE_2\left(\frac{r_{p2}}{R_l} + \mu_2 + 1\right) = \mu_2\, dE_1 + dE_i$$

from which the change in output potential dE_2 is given by

$$dE_2 = \frac{\mu_2\, dE_1 + dE_i}{\mu_2 + 1 + r_{p2}/R_l} \tag{7-61}$$

To relate dE_1 to dE_2 and dE_i, an examination is made of the operation of $T1$. It should be observed that dE_1 is the resultant of two effects. One is an increase which arises from the change in input potential and is $+dE_i \frac{r_{p1}}{R_3 + r_{p1}}$. This is the effect of the series circuit comprising R_3 and the tube $T1$ in series. The comparison potential source E_s does not appear, as it is assumed to be of constant potential and zero internal impedance. The second component is the amplified effect of the input to $T1$. This is $dE_2 \frac{R_2}{R_1 + R_2} K_1$, where K_1 is the gain of $T1$. That is, the total effective change dE_1 is

$$dE_1 = \frac{r_{p1}}{R_3 + r_{p1}} dE_i + \frac{R_2}{R_1 + R_2} K_1 \, dE_2 \tag{7-62}$$

which is written as

$$dE_1 = \gamma \, dE_i + \beta K_1 \, dE_2 \tag{7-63}$$

where $\gamma \equiv \frac{r_{p1}}{R_3 + r_{p1}} \qquad K_1 = \frac{-\mu_1 R_3}{r_{p1} + R_3} \qquad \beta = \frac{R_2}{R_1 + R_2}$

Now combine Eq. (7-63) with Eq. (7-61). This gives

$$dE_2 = \frac{\mu_2(\gamma dE_i + \beta K_1 \, dE_2) + dE_i}{(\mu_2 + 1) + r_{p2}/R_l}$$

which then becomes

$$dE_2 = \frac{dE_i(\mu_2\gamma + 1)}{\mu_2(1 - \beta K_1) + 1 + r_{p2}/R_l}$$

The potential stabilization ratio S is given by

$$S = \frac{dE_i}{dE_2} = \frac{\mu_2(1 - \beta K_1) + 1 + r_{p2}/R_l}{\mu_2\gamma + 1} \tag{7-64}$$

which approximates under normal conditions to

$$S \doteq \frac{-\beta K_1 \mu_2}{\mu_2 \gamma + 1} \tag{7-65}$$

The quantity S gives a measure of the effectiveness with which the regulator compensates for changes in potential in the input. For a regulator with a single d-c amplifier stage, S may be of the order of 300 to 1,000. If an improved value for S is required in order to achieve an almost ripple-free output, it is necessary that the gain of the d-c amplifier be increased. This is most easily done by adding a second stage of d-c amplification. Such circuits will be considered below. With such circuits, a value of S of 25,000 is possible.

b. Varying Input Potential—R_3 Connected to the Output Side of the Regulator. Somewhat improved results are theoretically possible if the plate-load resistor R_3 of the d-c amplifier stage $T1$ is connected to the output side of the regulator instead of the input side. That an improvement appears possible follows from the fact that one may now assume that the plate potential to which the amplifier $T1$ is connected is substantially constant and that the total change dE_1 is just the output from this stage, viz.,

$$dE_1 = \beta K_1 \, dE_2 \tag{7-66}$$

All other conditions remain as in the foregoing analysis. As a result, the term involving γ does not appear, and the corresponding stabilization ratio becomes

$$S = \frac{dE_i}{dE_2} = \mu_2(1 - \beta K_1) + 1 + r_{p2}/R_l \tag{7-67}$$

which approximates under normal conditions to

$$S = -\beta K_1 \mu_2 \tag{7-68}$$

Owing to the denominator that appears in Eq. (7-65), the value of S appears to be higher in Eq. (7-68). Actually, however, the d-c amplifier $T1$ operates in a more linear manner under condition a, with somewhat higher gain K_1. Under normal circumstances, the two connections yield about equal results, and both are used.

c. Varying Load. Suppose now that the change in the output potential when R_3 is connected as in case b results from a change in the load current, owing to the internal resistance of the supply. If this change in potential is again denoted as dE_2, then, as before, the potential appearing at the grid of the current control tube is

$$e_{g2} = dE_1 - dE_2$$

with

$$dE_1 = \beta K_1 \, dE_2$$

If g_m denotes the mutual transconductance of the series control tube, the resulting change in current through this tube due to a change in potential e_{g2} at the grid is

$$dI_p = g_{m2} e_{g2}$$

which is

$$dI_p = g_{m2}(1 - \beta K_1) \, dE_2 \tag{7-69}$$

The ratio of the change in output potential to the change in output current is denoted by R_0 and is the effective internal resistance of the regulated power supply. This is given by

$$R_0 = \frac{dE_2}{dI_p} \doteq \frac{1}{-g_{m2}\beta K_1} \tag{7-70}$$

In a typical case the effective internal resistance of the regulated supply may be as low as 0.5 ohm.

This calculation assumes, of course, that there is no change in the input applied potential with changes in output current, or, equivalently, that the internal impedance of the unregulated power supply is low. This condition is not often met in practice, and due account must be taken of this factor. If the internal resistance of the unregulated driving source is denoted by R_s, then by Eqs. (7-68) and (7-69) the total effective internal resistance of the regulated power supply is given by

$$R_0 = \frac{1}{-g_{m2}\beta K_1} + \frac{R_s}{S} = \frac{1}{-g_{m2}\beta K_1}\left(1 + \frac{R_s}{r_{p2}}\right) \qquad (7\text{-}71)$$

For a typical case $R_s = 500$ ohms, $S = 1{,}000$, so that the added resistance due to the regulation of the input source may be only about 0.5 ohm.

7-11. Design Consideration. A typical circuit that yields satisfactory results over a wide range of input potential and over a wide range of load current is given in Fig. 7-21. Although the diagram of Fig. 7-20 shows a triode as the d-c amplifier, it is found more desirable to use a

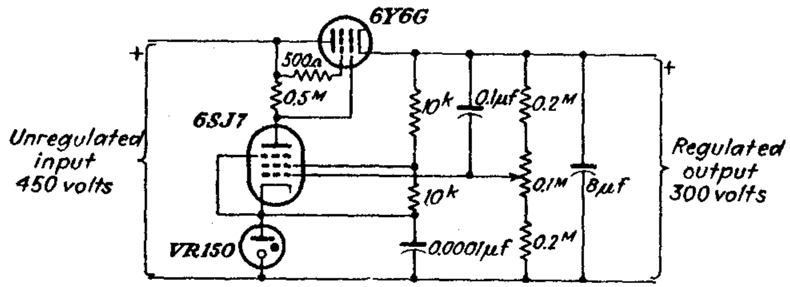

FIG. 7-21. An electronically regulated power supply.

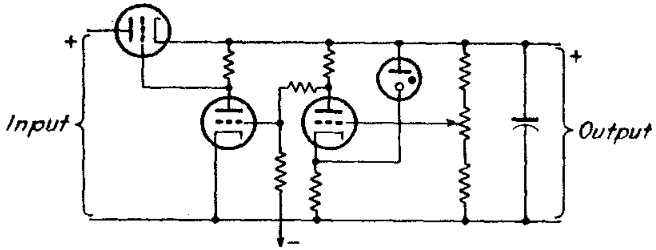

FIG. 7-22. An electronic potential regulator employing a two-stage d-c amplifier.

pentode in this position and this has been done in the circuit of Fig. 7-21. The reason for this is that it frequently happens that the d-c regulation is not as good as one would expect when a triode is used, primarily because the grid impedance of a high-gain triode is quite low. The grid impedance should be high, especially if the full gain capabilities are to be realized. A 6SJ7 tube is superior in this respect and hence is frequently used. The 6Y6G pentode called for in Fig. 7-21 gives satisfactory results. The 6AS7G tube possesses some advantages over this, since it has a relatively high tube current rating (125 ma) with a relatively low tube drop. Also, the heater-cathode insulation is sufficiently good to avoid the need for a separate filament heating transformer for this tube. The tube does have a rather low value of μ (= 2.1) which requires a rather large control potential.

If a potential regulator is required which is to provide a practically ripple-free output and an almost perfect regulation, it is necessary that the gain of the d-c amplifier be increased. This is most easily done by adding a second stage of d-c amplification to the regulator circuit. A variety of such circuits are possible, and several types are illustrated here. Figure 7-22 shows a simple two-stage resistance-coupled amplifier; Fig. 7-23 utilizes a cathode-coupled amplifier; and Fig. 7-24 uses a "cascode" amplifier.

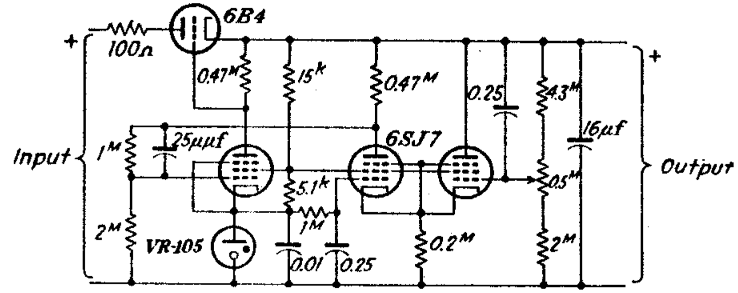

FIG. 7-23. A potential regulator employing a cathode-coupled amplifier.

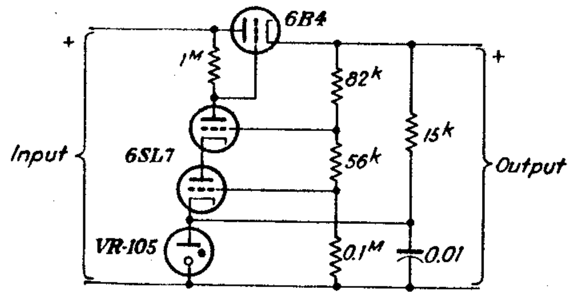

FIG. 7-24. A potential regulator employing a cascode amplifier.

7-12. Special Precautions.[6] Although the principles of operation of the regulated power supply are straightforward, it frequently happens that the maximum performance will not be realized in practice. It is well to consider some of the reasons for this. The important factors to be examined closely are the degenerative d-c amplifier loop and the various sources of hum.

a. The Degenerative D-C Amplifier Loop. For satisfactory operation, the d-c amplifier must be degenerative at all frequencies for which the loop gain is greater than unity. If this condition is not met, the system will oscillate (refer to Sec. 5-7) and the regulation properties will be greatly affected. One of the best ways to ensure that the power supply will not break into oscillation is to limit the h-f response of the amplifier. This is best done by including a large capacitor across the output (8 μf or 16 μf). This will usually provide the necessary h-f cutoff and will still keep the power-supply impedance low at high frequencies. The 0.1-μf capacitor from the grid of $T1$ to $B+$ serves to prevent a phase lag at the grid of the d-c amplifier and also compensates somewhat for additional phase shift in the amplifier and increases K_1 for frequencies above a cycle or two.

b. Sources of Hum. Among the sources of hum which give rise to a higher ripple in the output than is expected, and their possible cures, are the following:

1. Ripple from a-c heated filaments in the d-c amplifier. By grounding the center tap of the heater transformer and by choosing tubes with low heater-hum characteristics, the hum in the output potential can be reduced to 4 or 5 mv rms or less.

2. Ripple from common leads. This may arise from coupling between the d-c supply and some a-c source, such as a filament supply. The use of the chassis as a common ground with grounds to various parts of the chassis may introduce this hum potential. This effect is ordinarily small, perhaps several millivolts rms, except when the common coupling appears in the input of the d-c amplifier in the regulator, in which case it may be appreciable. To avoid this difficulty, grounds should be separately returned to a single point.

3. Ripple from supply potential. The screen potential to the d-c amplifier must be ripple-free. This may require a filter at the screen terminal at the tube base.

4. Ripple in the comparison potential source. It might be necessary to include a filter in the CR circuit for this purpose, in addition to the 0.0001-μf capacitor shown (which is to prevent any effects that might arise from the h-f plasma oscillations in the VR tube).

5. Induction loops. If coupling occurs between circuits by electrostatic or electromagnetic induction, it may be necessary to include a simple RC filter in the input circuit to the d-c amplifier and in the comparison-voltage circuit.

c. Heater Supply. When the heater of the d-c amplifier is fed from an unregulated source, changes in output with heater-potential changes may be quite noticeable. This can be eliminated by operating the heaters from the regulated d-c supply.